Model Order Reduction Techniques
For Real-time Parametric Crash and Safety Simulations

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1 Introduction

Model order reduction (ROM) techniques are interpolation methods exploiting exiting data sets (input and output) derived from an existing model or experimental setup. The starting point is a DOE-type design which covers as best as possible the design space (space filling property). Contrary to response surface (polynomial based) designs where the selection of design points at particular positions is important due to a-priori properties of the fitted surface, for ROM techniques, the most important issue is the space-filling capacity and sufficient “modal” representation of the response.

In some cases ROM can be considered as algebraic solutions, exploiting decomposition of time-dependent phenomena into special and temporal components, for reducing the volume of a data set while preserving the most important parts of the information contained within the data which is necessary for retrieving all or the most essential part of the data when needed. Else we can also consider ROM as clustering or other “lossy” efficient data compression techniques, allowing for the reduction of the amount of data required for the reconstruction of the complete data set. Both such techniques allow for creating on-board and real-time applications based on voluminous experimental or simulation results (ex. Finite element).

In this paper we shall present the major idea behind the reduction (or fusion) methods as well as providing three potential applications for crash and safety simulations. The results are obtained by ODYSSEE (Lunar) software [2] and compared to LS-DYNA FEM results.

*KEYWORDS: Model Reduction, ROM, POD, CLUSTERING, SVD, On-board computing, Real-time modeling, Crash, Safety, Parametric design

2 Reduced Order Modeling

In 2015 Kayvantash [1] reported on an innovative solution and presented results based on model reduction techniques in order to exploit fully the results of time dependent DOE’s such as in the case of Crash or ALE. An explanation of the algorithm (based on POD standing for Proper Orthogonal Decomposition) was given and a demonstration of the method was provided for a typical safety simulation model as well as an ALE application (ballistic impact including detonation and fluid/structure interaction). In 2017 Yasuki [5] presented a full sled test demonstration using solutions developed by Kayvantash [2] and provided detailed comparative studies of the FEM and ROM solutions. Further, in 2018 Yasuki [6] presented a population based comparison of 100 sled tests using an FE dummy + sled simulation and compared it to ROM reconstructions of the same scenario. Additionally, after validation of the comparative studies, a Pareto Front profile of Nij for 10000 hypothetic individuals was established. In 2018 Ovazza and Kayvantash [4] developed new methods of decomposition based on clustering techniques and machine learning which improved yet further the computation speed and reduced the data storage required for implementing ROM on on-board devices. These works have launched an interest in applications of ROM for crash and safety simulations, parametric studies and optimizations. In this work we shall explain the principles and discover further the potential of such reduction methods.

Some reduction techniques are based on reducing the order of the operators representing the set of algebraic operations resulting from discretization of differential equations. These are commonly referred to as PGD (Proper Generalized Decomposition) and are directly applicable to PDE solvers.

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These solutions are considered as “intrusive” since they require extensive modifications or even rewriting of solver code. However, recent research has shown that using machine learning techniques other physical phenomena - for which a PDE (partial differential equation) is not known, or does not exist, or not fully or efficiently handled from a computation effort point of view - may also be explored and predicted using ROM methods. In particular, “non-intrusive” or a posteriori techniques of results post-processing are of interest. These methods (such as POD) require simply the establishment of a new vector base (orthogonal or reference vector as opposition to the original Cartesian-temporal reference axes) and a projection of the known model or experimental results on that new base. This technique is similar to a PCA (Principal Component Analysis) projections.

Independent of the method selected, a ROM technique implies three steps: Decomposition, Reduction and Reconstruction. In the next paragraphs we shall describe shortly these steps and then proceed with a few crash and safety related applications.

In what follows we shall use the following conventions:

\[ X = \text{Table of design parameters (or variables) defining experiments (we may also call this a DOE)} \]

\[ Y = \text{Results of the experiments associated with } X \]

\[ XN = \text{A new table of parameters for which we seek results without actually performing experimenting or computing} \]

\[ YN = \text{Results of the new experiments associated with } XN \text{ (to be computed by ROM)} \]

\[ G, H, S = \text{Decomposition matrices of which } G_r, H_r, S_r \text{ are sub-matrices.} \]

\[ y = \text{Spatial or latent metrics of system (sometime called coefficients)} \]

\[ t = \text{time} \]

\[ m, n, p, r = \text{indices (dimensions of matrix)} \]

Note – The N in \((XN \text{ and } YN)\) stands for NEW since we do not include \(XN\), which is known, in the decomposition process and we don’t know the \(YN\) (since we want to predict it).

2.1 Decomposition

Assuming we have a set of data representing a time dependent phenomena such as \(Y(y, t)\), where \(y\) represent a “spatial” but latent variables associated to the response.

We define the decomposition:

\[ Y(y, t) = G(y) \cdot s \cdot H(t) \quad \text{where } G \text{ is a “spatial” operator (a matrix) , } s \text{ is a transfer matrix (diagonal or semi-diagonal) and } H \text{ is a “modal” operator (a matrix). Indeed a popular method often used at this phase is called Singular Value Decomposition in which case the } s \text{ matrix includes the singular values of } Y. \]

We may obtain similar decomposition via an appropriate clustering method such that:

\[ Y(y, t) = \sum \alpha \cdot \Psi(y) \]

Where \(\Psi\) are “weights or cluster functions and \(\alpha\) are coefficients both obtained via an appropriate learning or clustering algorithm (Support Vector Machines, PCA, MLP, etc.).

In both cases we can claim to have decomposed the original data set from the physical or Cartesian reference frame on to a new set of basis with special and very useful properties such as orthogonality (in case of SVD decomposition) or compactness (in case of clustering).

As common in most supervised techniques, let’s assume also that we know of another set of data \(X\) (representing a sampling table of some properties or criteria which have resulted in the variations of the responses. In short, \(X\) represents a Design of Experiments and \(Y\) the corresponding outcome at the \(X\) sampling points. We are interested in establishing a function (the ROM operator) which relates \(Y\) to \(X\) as in the case of a “supervised” learning algorithm.
In particular by applying the decomposition and the projection we have separated the variables \( y \) and \( t \) (similar to modal decomposition techniques in transient dynamic problems). We can claim to have obtained a decoupled version of the original set of data, in another vector basis other than the original Cartesian (or physical) basis. Secondly, we can now perform separate operations on any of the two major components (matrices \( G \) or \( H \) or \( \alpha \) or \( \Psi \)) in order to further exploit the data, in association with many available interpolation techniques.

### 2.2 Reduction

The primary objective of all ROM techniques is to compress or reduce the volume of data to be handled and in consequence, the number of floating point operations required for the response prediction. The purpose of this compression and reduction is to allow for real-time computation providing reliable parametric studies based on existing DOE’s.

In general matrix \( s \) contains a set of singular values (if SVD based decomposition is performed) of descending order, corresponding to the effect of the corresponding singular value on the complete reconstruction of original base. This is very similar to the approach used in modal analysis where only lower (or higher) modes are retained. If all non-zero components of \( s \) are used for the reconstruction (see next phase in 2.3), we can claim a "non-lossy" reconstruction whereas an abandon of smaller singular values can lead to a "lossy" form of the reconstruction compression.

Similar argumentation holds for a clustering based decomposition, where coefficients \( \alpha \) and weights matrices \( \Psi \) may be maintained or removed from the reconstruction phase due to the importance (often called " inertia") representing contribution to the overall information content of the data base.

This results in using only subsets of \( G \) and \( H \) (or \( \alpha \) and \( \Psi \)) based on some tolerance criteria is simply a compression (fusion) of the original data base.

In this case only the "reduced" matrices need be considered with important consequences for CPU and storage issues which are essential for on-board computing solutions. In matrix form, if the original data \( X \) and \( Y \) are of dimensions \( \mathbf{X}_{m,n} \) and \( \mathbf{Y}_{m,p} \) (assuming \( p>>m \) which is reasonable for time responses,) then the decomposed matrices may have the dimensions \( G_{m,r} \), \( S_{r,r} \) and \( H_{r,p} \) (where \( r \) represents the number of retained modes or clusters). We can observe that if a new set of parameters is experimented such as \( \mathbf{X}_{N_{m,n}} \) the reduced matrices still provide a result matrix \( \mathbf{Y}_{N_{m,p}} \) (same order as \( \mathbf{Y}_{m,p} \)).

At any instance we can either reconstruct the original set of data either completely (using decomposed operators) without loss or partially (using reduced operators) with some loss of information. A partial reconstruction would mean that we only retain selected parts of \( G \) and \( H \), a technique which can be assimilated to filtering or lossy compression. This allows a reduction of the volume of the "on-board" data as well as a filtering of noise or other high frequency sources of uncertainty in the results.

### 2.3 Reconstruction

In what follows we shall only refer to the SVD based approach (POD) since the computational steps are identical for clustering based solution.

In case of predictions which require the results \( \textbf{YN} \) for a new parameter set \( \textbf{XN} \), we observe that we can replace \( G \) or \( H \), by their modified updates \( G' \) or \( H' \), taking into account of the change in the parameters, and obtain a new set of response corresponding to slightly modified new positions \( XN \) compared to the original \( X \). In simple terms we can make predictions of effect of \( X \) moving to \( XN \) on \( Y \) (moving to \( YN \)) via considering its effect on \( G \), moving to \( G' \) or \( H \) moving to \( H' \) only. If we need to compute \( YN \), we need to compute the effect of \( XN \) on \( G \) or \( H \). It turns out that the updates \( G' \) or \( H' \) may be simply obtained by an adequate interpolation technique such as radial basis functions, kriging, etc. A final back multiplication provides the results \( YN \).
Lastly, it is important to point out that it is possible to mix the experimental and numerical results and construct a unified data base \( \{ \text{Y}_{\text{experiments}} + \text{simulation} \} \) in which case we could call the data base a fusion of real and virtual information. This alone opens new horizons in modelling which until now was always suffering from “correlation” and “validation” issues. We can claim that we have a new tool which benefits from the best of both experimental and numerical technologies.

3 Applications

Three applications will be presented hereafter showing different potential of the proposed method and solution software. The starting point is, as in many other parametric design projects, the construction of the DOE samples and the time dependent response of the system for each case present in the DOE table (Figure 1.). Note that the response may be measured experimentally or computed via discretization techniques such as Finite Elements, etc.

For all three applications a Desktop version of the software ODYSSEE (Lunar) has been used running on a Laptop (DELL M4800, INTEL i7 4910 MQ, 2.9GHz, 2.9GHz, 16 GB RAM) was used.

3.1 Simply supported plate

A simply supported plate with variable thickness and moving support position is considered in order to compute the deflection at the loaded side of the plate (Figure 2.). A DOE of 15 cases was first assumed and analysed via FEM. The results were obtained for four new points by ROM methodology (Figure 3.) A finite element based solution (1000 elements) is compared with a real-time (instantaneous, \( \approx 1 \text{sec} \)) solution. CADLM’s ODYSSEE (Lunar) software [2] was used for the ROM modelling.

3.2 Simplified Bonnet head impact without or with rupture

A simply supported plate with a spherical impactor is considered as a simple model of a head impact on a bonnet as in the case of pedestrian impact. The plate is made of composite material allowing for failure corresponding to rupture of fibres (Figure 4.). It is intended to evaluate the capabilities of the ROM solution in case where rupture (or bifurcation occurs). This clearly shows that the method is not limited to simple, linear or smoothly non-linear cases. A finite element based solution (1000 elements) is compared with a real-time (instantaneous, \( \approx 1 \text{sec} \)) solution. CADLM’s ODYSSEE (Lunar) [2] [3] was used for the ROM modelling as well as conducting the DOE runs.

3.3 Sled with airbag and uncertain impact velocity

A typical sled test is considered. An FEM model is constructed using LS-DYNA dummy and the results of the FEM model are obtained for a DOE of 9 Optimal Latin Hyper Cube augmented by 6 additional “space filling” type samples. A DOE of 15 cases was first constructed and the results were computed via LS-DYNA. CADLM’s ODYSSEE (Lunar) software was used [2]. The results of the pelvis acceleration and chest displacement are compared (Figure 5. & Figure 6.) Additional parametric studies were also conducted and animation screen shots are also obtained and compared.

4 Figures and Tables
**Fig. 1:** Procedure for ROM modeling (using ODYSSEE (Lunar) Software).

**Fig. 2:** Computing influence lines for moving support point – Black square points represent the DOE samples and the red circles represent the prediction points.
Fig. 3: Computing influence lines for moving support point at four combinations of model parameters (colored circle points) – Results (Finite Elements versus ROM)

Fig. 4: Plate-sphere impact (representing bonnet-head in pedestrian case) including possible rupture of composite material – Results (Finite Elements versus ROM)
Model Parameters
- Gas injection
- Impact velocity solpes

DOE of 15 Samples

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15 Output:
- Thorax compression
- Head acceleration,
- Pelvis acceleration,

Fig.5: Sled test + airbag with variations on airbag mass injection and impact velocity - DOE with 3 parameters, 9 (OLH) + 6 "space filling" samples

Population studies
Parameter sensitivity

Lunar
Data fusion & model reduction

Lunar: 2 s  FEM: 1 hour
Fig.6: Results of sled test comparing FEM (LS-DYNA) and ROM (ODYSSEE/Lunar). ROM method may be used with simplicity for inverse optimization (model fitting) as well as sensitivity and population studies. (1)

5 Summary and conclusions

All three comparative studies undertaken during this work were used to compare results accuracy and computing time. All the results were satisfactory in terms of differences with the FEM counterparts (difference in the range of 1-10%). All the ROM CPU times were at most in the order of a few seconds.

Results obtained by ROM lie within 1-10% of the finite element counterparts. The computing time for ROM models is often in the order of seconds whereas the FE models range from minutes of computing (applications 1 & 2) to hours for sled test model. In practice any new set of parameters could be studied in a matter of seconds or “quasi real-time” with a gain of many orders of magnitude in terms of computation time.

Finally, comparative performance and portability studies were also investigated on an on-board version (Raspberry PI 3) of ODYSSEE/Quasar (The results will be published in a separate work). It has been shown that nearly all the presented applications may be conducted on a very small CPU while the current limitations concern the storage available on such on-board devices.

6 Literature